SOLVE $y'' + 6y' + 9y = 0$, $y(0) = -1$, $y'(0) = 6$

$$\mathcal{L}\{y'' + 6y' + 9y\} = \mathcal{L}\{0\} = 0 \qquad Y = \mathcal{L}\{y\}$$

$$s^2 Y - sy(0) - y'(0) + 6(sY - y(0)) + 9Y = 0$$

$$s^2 Y + s - 6 + 6(sY + 1) + 9Y = 0$$

$$(s^2 + 6s + 9)Y = -s + 6 - 6 = -s$$

$$\mathcal{L}\{y\} = \frac{-s}{(s+3)^2} = \frac{\overset{-1}{\cancel{A}}}{s+3} + \frac{\overset{3}{\cancel{B}}}{(s+3)^2}$$

$$y = -e^{-3t} + 3te^{-3t}$$

$$\frac{-s}{(s+3)^2} = \frac{A}{s+3} + \frac{B}{(s+3)^2} \longrightarrow -s = A(s+3) + B$$

$s = -3$: $3 = B$

COEF OF s: $-1 = A$

MANDATORY SANITY CHECK: $s = -2$ $\quad \frac{2}{1^2} = 2$

$$\frac{-1}{1} + \frac{3}{1^2} = -1 + 3 = 2 \checkmark$$

SOLVE $w'' + w = t^2 + 2 \quad w(0) = 1, \; w'(0) = -1$

$$\mathcal{L}\{w'' + w\} = \mathcal{L}\{t^2 + 2\}$$

$$s^2 W - s\overset{1}{\cancel{w(0)}} \overset{+1}{\cancel{- w'(0)}} + W = \frac{2}{s^3} + \frac{2}{s}$$

$$(s^2 + 1) W = \underline{s - 1 + \frac{2}{s^3} + \frac{2}{s}}$$

$$W = \frac{s - 1 + \frac{2}{s^3} + \frac{2}{s}}{s^2 + 1}$$

$$W = \frac{s-1}{s^2+1} + \frac{\frac{2}{s^3} + \frac{2}{s}}{s^2+1} \cdot \frac{s^3}{s^3}$$

$$= \frac{s-1}{s^2+1} + \frac{2 + 2s^2}{(s^2+1)s^3}$$

$$= \frac{s-1}{s^2+1} + \frac{2\cancel{(1+s^2)}}{\cancel{(s^2+1)}s^3}$$

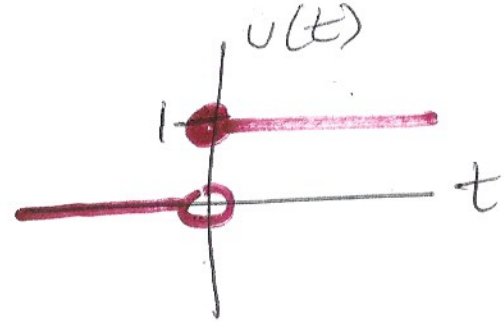
$$= \frac{s}{s^2+1} - \frac{1}{s^2+1} + \frac{2}{s^3}$$

$$w = \cos t - \sin t + t^2$$

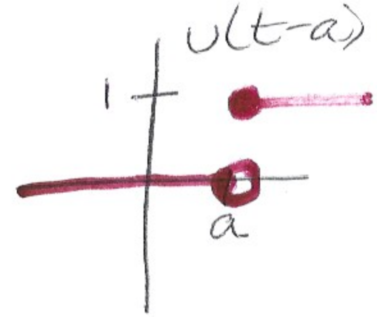
7.3

HEAVISIDE FUNCTION

$$u(t) = \begin{cases} 0, & \text{IF } t < 0 \\ 1, & \text{IF } t \geq 0 \end{cases}$$



$$\mathcal{L}\{u(t)\} = \int_0^\infty e^{-st} u(t)\,dt$$
$$= \int_0^\infty e^{-st} \cdot 1\,dt = \mathcal{L}\{1\} = \frac{1}{s}$$



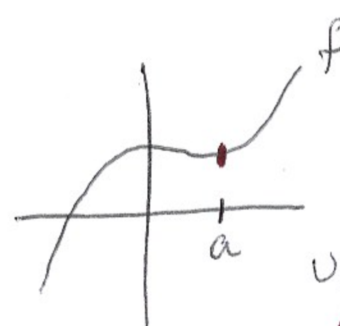
$u(t-a)$ IS $u(t)$ SHIFTED RIGHT BY a

$a > 0$

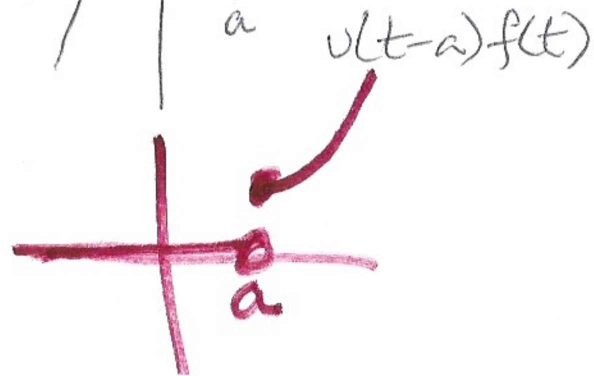
$$u(t-a) = \begin{cases} 0, & \text{IF } t-a < 0 \\ 1, & \text{IF } t-a \geq 0 \end{cases}$$

$$u(t-a) = \begin{cases} 0, & \text{IF } t < a \\ 1, & \text{IF } t \geq a \end{cases}$$

WHAT IS $u(t-a)f(t)$?



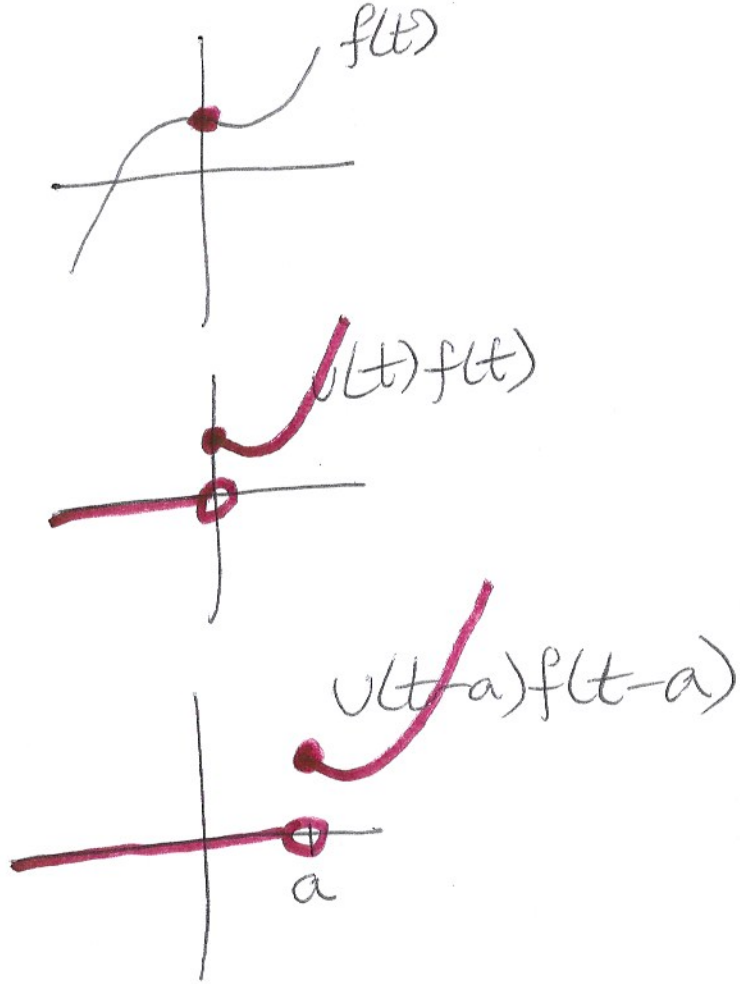
$$u(t-a)f(t) = \begin{cases} 0 \cdot f(t), & \text{IF } t < a \\ 1 \cdot f(t), & \text{IF } t \geq a \end{cases}$$
$$= \begin{cases} 0, & \text{IF } t < a \\ f(t), & \text{IF } t \geq a \end{cases}$$



WHAT IS $u(t-a)f(t-a)$?

$u(t)f(t)$ SHIFTED RIGHT BY a

$= u(t-0)f(t)$



$$u(t-a)f(t) + u(t-b)g(t) \qquad \text{IF } b > a > 0$$

$$= \begin{cases} u(t-a)f(t) + u(t-b)g(t), & t < a \\ u(t-a)f(t) + u(t-b)g(t), & a < t < b \\ u(t-a)f(t) + u(t-b)g(t), & t > b \end{cases}$$

$$= \begin{cases} 0\,f(t) + 0 \cdot g(t), & t < a \\ 1 \cdot f(t) + 0 \cdot g(t), & a < t < b \\ 1 \cdot f(t) + 1 \cdot g(t), & t > b \end{cases}$$

$$= \begin{cases} 0 & , t < a \\ f(t) & , a < t < b \\ f(t) + g(t), & t > b \end{cases}$$

eg. $u(t-2)t^2 + u(t-5)e^t$

$$= \begin{cases} 0 & , t < 2 \\ t^2 & , 2 < t < 5 \\ t^2 + e^t & , t > 5 \end{cases}$$

WRITE

$$k(t) = \begin{cases} 0 & , t < \pi \\ \overset{f(t)}{\cos t} & , \pi < t < 4\pi \\ \overset{f(t)+g(t)}{\sin t} & , t > 4\pi \end{cases}$$

IN HEAVISIDE NOTATION

$$u(t-\pi)\cos t + u(t-4\pi)(\sin t - \cos t)$$

WRITE

$$m(t) = \begin{cases} 0, & t < 2 \\ t^2 \xleftarrow{} f(t), & 2 < t < 5 \\ t^3 + 2t^2 \xleftarrow{} f(t)+g(t), & 5 < t < 6 \\ t^4 - 2t^3 \xleftarrow{} f(t)+g(t)+h(t), & t > 6 \end{cases}$$

IN HEAVISIDE NOTATION

$$u(t-2)\, t^2 + u(t-5)(t^3+2t^2-t^2) + u(t-6)(t^4-2t^3-(t^3+2t^2))$$

$$= u(t-2)t^2 + u(t-5)(t^3+t^2) + u(t-6)(t^4-3t^3-2t^2)$$

$$\mathcal{L}\{u(t-a)f(t)\} = \int_0^\infty e^{-st}\, u(t-a) f(t)\, dt$$

$$= \int_a^\infty e^{-st} f(t)\, dt$$

LET $v = t - a$, $t = v + a$, $dt = dv$

$$= \int_0^\infty e^{-s(v+a)} f(v+a)\, dv$$